



## Appearances Can Be Deceiving!

### Overview:

Students explore the relationship between angular width, actual size, and distance by using their finger, thumb and fist as a unit of angular measurement in this hands-on activity.

**Target Grade Level: 6-8**

**Estimated Duration: 2 class periods or about 90 minutes**

**Learning Goals: Students will be able to...**

- relate angular width to actual size using distance.
- quantify the relationship between angular width and distance.
- understand why the Sun appears smaller from Pluto than from Earth.

### Standards Addressed:

#### **Benchmarks (AAAS, 1993)**

The Nature of Mathematics, 2C: Mathematical Inquiry

The Mathematical World, 9B: Symbolic Relationships

Common Themes, 11B: Models

#### **National Science Education Standards (NRC, 1996)**

Science as Inquiry, Standard A: Abilities necessary to do scientific inquiry

Earth and Space Science, Standard D: Earth in the solar system

History and Nature of Science, Standard G: Nature of science

#### **Principles and Standards for School Mathematics (NCTM, 2000)**

**Measurement Standard:** Understand measurable attributes of objects and the units, systems, and processes of measurement; Apply appropriate techniques, tools, and formulas to determine measurements

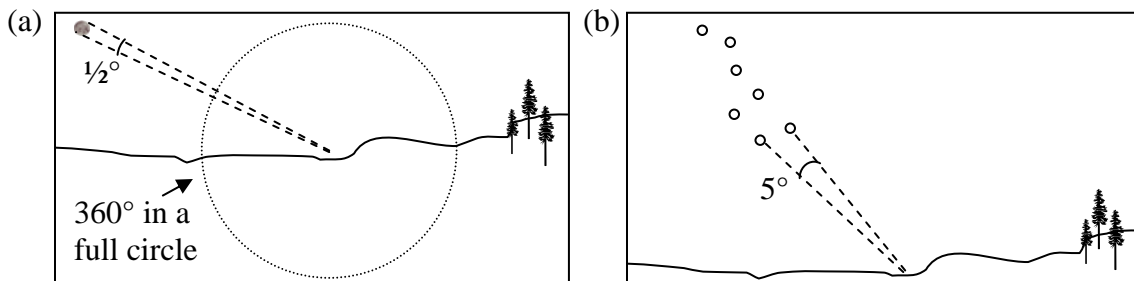
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## Background:

Here on Earth we can often determine the size of objects or the distance between objects using rulers or measuring tapes. Astronomers do not have that luxury, since they can rarely visit the object they wish to measure. One solution to this is a system of measurement that is based on angles. It employs the degree ( $^{\circ}$ ) as a fundamental unit, with  $360^{\circ}$  in a full circle (Fig. 1).

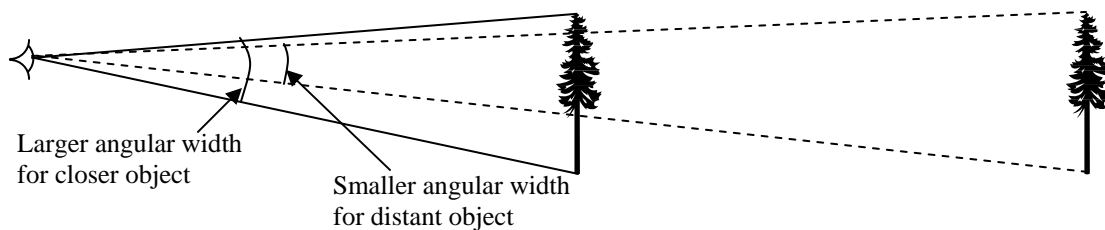
Angular measurements are used to indicate the apparent size of an object in the sky. The apparent size of an object is simply the size it appears to be. It therefore depends on the object's actual size and its distance from the observer. Astronomers use angular measurements to quantify an object's apparent size. For example, we know the full Moon appears quite small when viewed from Earth; to quantify how small it appears, we say it has an angular diameter of about  $\frac{1}{2}^{\circ}$  (Fig. 1). Of course the apparent or angular diameter is very different from the Moon's actual diameter, which is 3,476 km.



**Figure 1.** An example of the system of measurement based on angles in degrees, with 360 degrees in a full circle or about  $180^{\circ}$  from horizon to horizon. (a) The full moon, when observed from Earth, has an angular width or diameter of about  $\frac{1}{2}^{\circ}$ . (b) Two stars in the Big Dipper are separated by an angular distance of  $5^{\circ}$ .

Here are two ways to think about the relationship between angular width and actual size:

1. For an object of a given actual size, its apparent (angular) size decreases as you move away from it. For example, if you are standing next to a mature tree it seems large, whereas if you are standing 100 yards from it, the tree appears smaller. If you move to 200 yards from it, the tree will appear smaller still. Finally you will reach a distance where the tree appears so small it cannot be distinguished from the surrounding landscape.











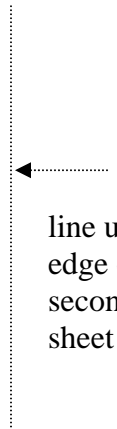
START



1°



2°



line up  
edge of  
second  
sheet here

3°



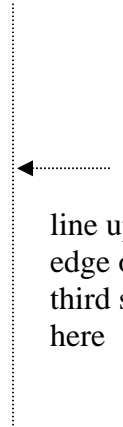
4°



5°



6°



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edge of  
third sheet  
here



7°



8°



9°



←  
line up  
edge of  
fourth  
sheet here

10°



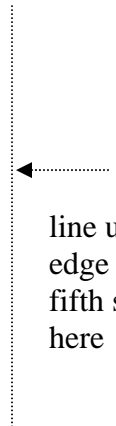
11°



12°



13°



line up  
edge of  
fifth sheet  
here

14°



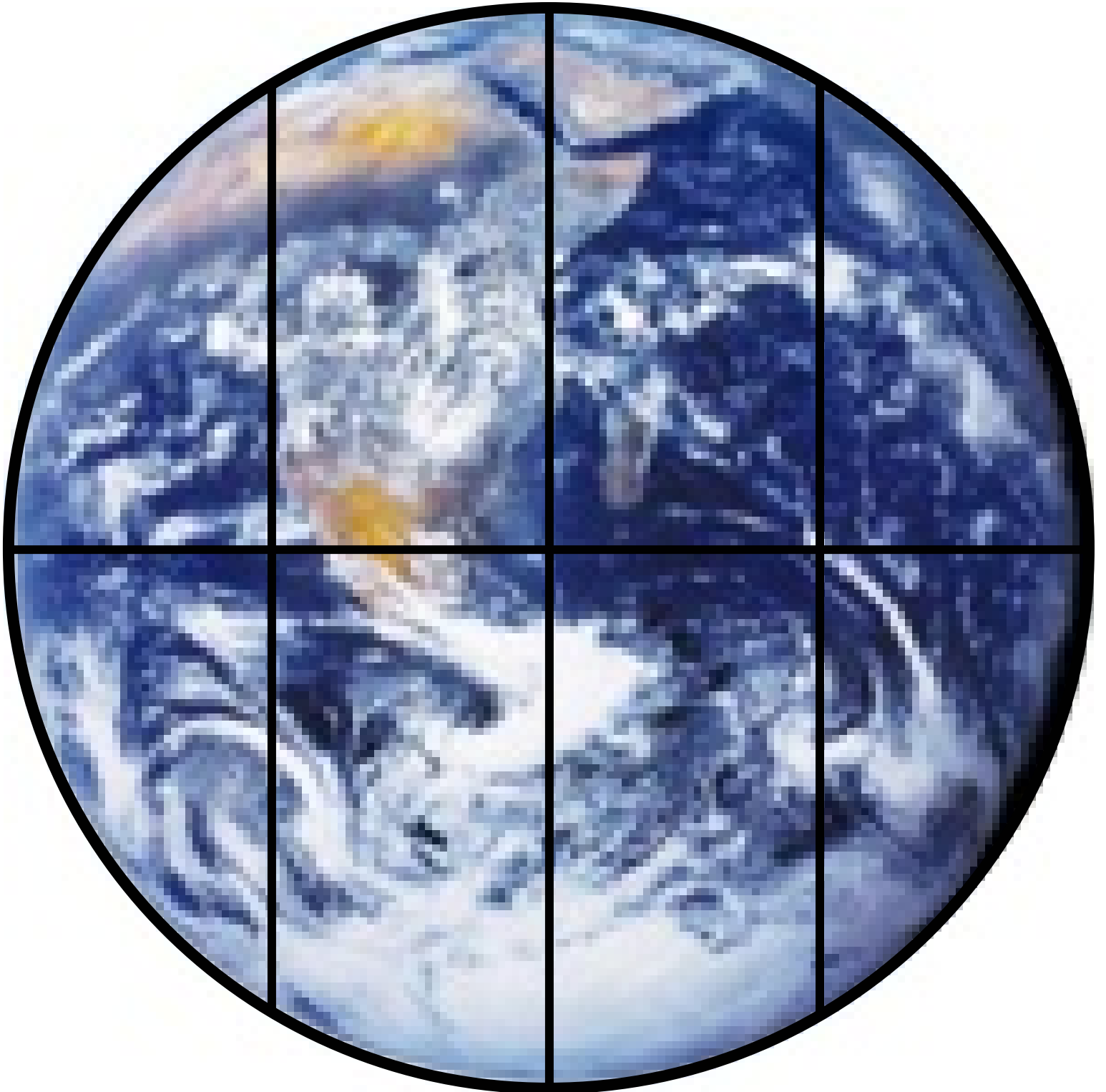
15°



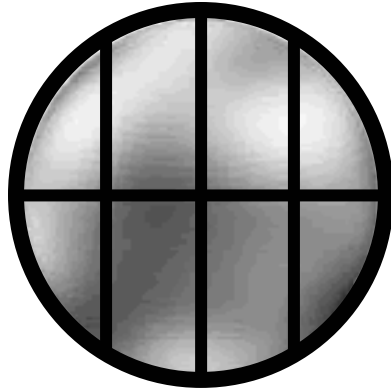
16°



# Earth



# Pluto



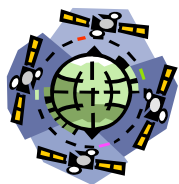
## Activity: Appearances Can Be Deceiving!

Name:

Today you are going to explore a measuring tool that you always carry with you. It is the very same tool you use to write, and the same tool you use to drink a glass of water. But did you know you could use your hand as a measuring tool?

Here is how it works: First notice that your teacher has placed a Degree Ruler and some planet diagrams on the board or wall. Also, on the floor are pieces of tape indicating the distances from the objects on the board or wall. You are going to determine the angular width of your finger, thumb, and fist using the Degree Ruler and the angular width of the diagrams of Pluto and the Earth while standing at these pieces of tape.

Proper technique is VERY important! Always hold your arm out straight in front of you at eye level and extend either your pointer finger, thumb or make a fist. You will be working with a partner. One of you will take a measurement while the other records the data, and then you will switch roles. You will visit all three stations before going to the ‘Observations’ and ‘Calculations’ sections.



### Data Collection

**Step 1.** At **STATION 1** you will determine the width or angular width of your finger, thumb, and fist in degrees, and record the data below. Begin by extending your arm out straight in front of you at eye level. Extend your index finger (“pointer finger”) straight up. Without bending your arm, line up the left edge of your finger with the left side of the Degree Ruler (marked with “START HERE”). Determine how many degrees your finger covers—that is, how wide it is in degrees.

Distance from the Degree Ruler: \_\_\_\_\_ meters

	Finger	Thumb	Fist
Angle measure:			

**Step 2.** At **STATION 2** you will see a diagram of Earth and a diagram of Pluto on the wall, with three pieces of tape on the floor labeled with the distance from the diagram. The planet diagrams use the same relative scale for the size of the planets. This means that it looks as if you were standing the same distance from each of them. While standing at each piece of tape, you should now measure the angular width of each planet using your finger, thumb, and fist. Remember to hold your arm out straight in front of you! Record your measurements below:

Distance from the planet diagrams: \_\_\_\_\_ meters

	Fingers	Thumbs	Fists
Earth:			
Pluto:			

Distance from the planet diagrams: \_\_\_\_\_ meters

	Fingers	Thumbs	Fists
Earth:			
Pluto:			

Distance from the planet diagrams: \_\_\_\_\_ meters

	Fingers	Thumbs	Fists
Earth:			
Pluto:			

**Step 3.** At **STATION 3** you will see the Earth and Pluto planet diagrams on the wall. Choose one of the planets, and determine how far away from the diagram you must stand so that it is exactly one finger wide, exactly two fingers wide, and exactly three fingers wide. Mark the distances on the floor with a piece of tape, and measure them using a meter stick. Record your distances below:

Planet chosen: \_\_\_\_\_

1 finger wide = \_\_\_\_\_ meters

2 fingers wide = \_\_\_\_\_ meters

3 fingers wide = \_\_\_\_\_ meters



## Observations

- In step 2 you measured the angular widths of the planets in units of ‘number of fingers, thumbs, and fists’. Compare your values with your partners. Are they exactly the same or are they different? Also compare your values with the values of another group. Are all of the values the same or do they differ?



a) If your measurements from step 2 are different from your partners or from another group's measurements, does it mean the size of the planets changed? What is different?

b) If people have different answers for step 2, is one wrong and another right? Why or why not?

6. In step 2 you measured the angular width of Earth from three distances. In the first row of the table below, record the three different distances from which Earth was measured in order of increasing distance. In the second row of the table, record the angular width of Earth (measured in fingers) at the corresponding distance.

	Closest to Earth	Middle	Farthest from Earth
Distances, in order of closest to farthest:			
Angular width, in fingers, corresponding to the distance in the box above it:			

a) In the table above, as the distance increases (i.e. as you move farther from the diagram of Earth) what happens to the angular width of Earth, as measured in 'fingers'?

b) What causes the angular width of the Earth to change?

c) Instead of looking at planets, imagine you were to look at a car parked directly in front of you, and then at the same car parked on the far side of a large parking lot. In which location would the car appear larger?

d) In c), does the actual size of the car change?

e) In c), what changes? Would this change cause the angular width of the car to change?

7. Read the following dialogue between two students, and answer the associated questions below. Keep in mind that the apparent width is how big or small the object appears to be.

Student 1: “So I guess the apparent width of an object only depends on the actual size of an object.”

Student 2: “But I thought the apparent width of an object only depends on how far away you are from it—the distance between you and the object.”

a) After reading the conversation between the two students, who do you think is right? (select 1)

- i. Student 1
- ii. Student 2
- iii. Both students
- iv. Neither student

b) If one or both of the students are incorrect, modify their statements to make them both correct here.



## Calculations

8. In step 2 you measured the angular widths of the planets at 3 different distances.
- a) Record those distances and the associated angular widths in fingers in the following data tables:

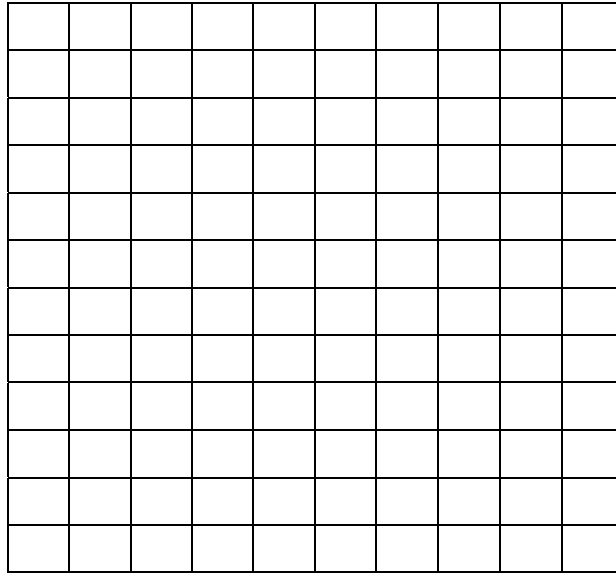
Pluto	
Distance (m)	Angular width (fingers)
1.	
2.	
3.	

Earth	
Distance (m)	Angular width (fingers)
1.	
2.	
3.	

- b) Plot the above data below, and draw a line through the data points:

### Pluto' s Angular Width

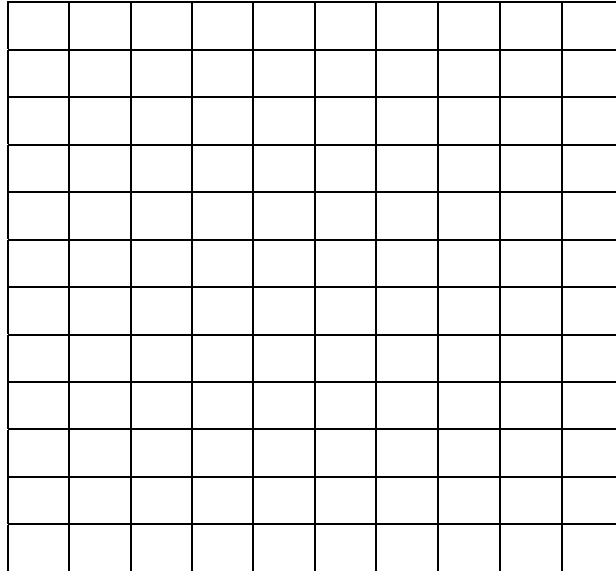
Angular width  
(fingers)



Distance (meters)

### Earth' s Angular Width

Angular width  
(fingers)



Distance (meters)

c) Compare the two graphs you created. How are they similar?

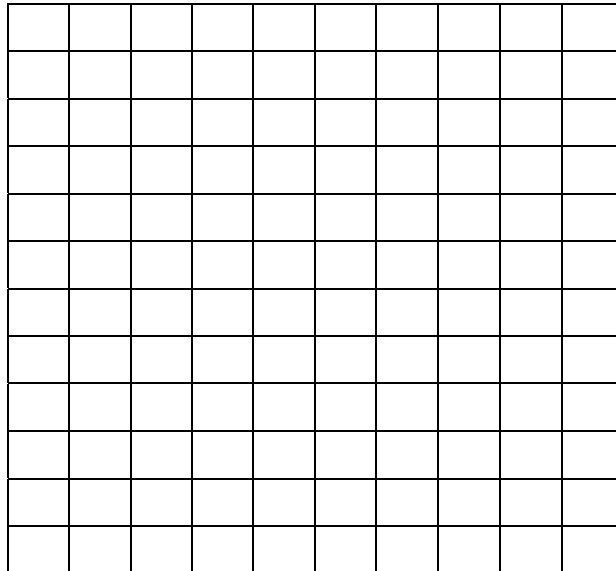
d) Describe the trend or pattern you observe in the graphs in your own words.

9. Refer to step 3. Record your values in the table, and plot the data in the graph provided.

Planet: _____	
Distance (m)	Angular width (number of fingers)
1.	1
2.	2
3.	3

Angular Width of Planet: \_\_\_\_\_

Angular width  
(fingers)



Distance (meters)

- a) Compare the graph here with the two graphs you created in number 8. What are the similarities and differences? Describe any relationship you observe between distance and angular width.

10. Refer back to step 1 (in Data Collection section) for the following values:

Record the distance you were standing away from the Degree Ruler here: \_\_\_\_\_

Record the angular width of your thumb in degrees in column two, below (in units of ‘degrees/thumb’ ).

Refer back to step 2 for the following values:

Record the angular widths of Pluto and Earth in ‘number of thumbs’ in column 1, below.

	Thumbs *	Degrees/thumb =	Degrees
Earth:			
Pluto:			

Calculate the number of degrees in the diagrams of Earth and Pluto based on your distance and the angular width of your thumb in column 3 above.

11. Throughout this exercise you have used the angular width in degrees of your finger, thumb and fist to determine the angular width of diagrams of planets. However, this tool is not limited to diagrams of planets in the classroom. You can use these values to observe the number of degrees between stars in the sky or in the angular width of the full moon!

**HOMEWORK:**

Extending your arm out straight in front of you, use the angular width of your finger in degrees to determine the angular width of the full moon in degrees. This is called the Moon's 'angular diameter'. How does the angular diameter relate to the Moon's actual diameter?

## Extensions and Adaptations:

- If you do not have enough space for three stations, you can simply take down the Degree Ruler once all students have finished step 1, and replace it with a diagram of Earth and of Pluto side-by-side, although this will take longer.
- If you would like to use a distance other than 4 meters to calibrate students' fingers, thumbs, and fists as units of measurements, you will need to make a different Degree Ruler, as follows. Substitute your distance  $r$ , in meters, in place of 4 m to calculate the new centimeters per degree, following this example:
  - i. circumference,  $c = 2 \pi r$
  - ii. since there are  $360^\circ$  in a circle,  $c / 360^\circ =$  your increment in meters
  - iii. your increment in meters per degree \* 100 = your increment in cm per degreeExample using  $r = 4$  m:
  - i.  $c = (2)(3.14\dots)(4 \text{ m}) = 25.1327 \text{ m}$
  - ii.  $25.1327 \text{ m} / 360^\circ = 0.0698 \text{ m/degree}$
  - iii.  $0.0698 \text{ m/degree} * 100 =$  about 7 cm in one degree

- Then you would have to make a new ruler with your increment in centimeters between every line or degree, as in the Degree Ruler provided.
- Simplification for lower grades: To modify this activity for lower grade levels...
  - Divide class into two groups. Ask students to form two lines with the groups facing each other (like in 'red-rover'). Pairs are formed between the student in group 1 and the student in group 2 that are facing each other. Or students in both rows can count off, and like numbers become pairs.
  - One row remains stationary with the each student in the other row moves to a distance where his/her two thumbs—held at eye-level, with both arms fully extended and thumbs pressed together—just covers his/her partner's face. They note this distance using some unit of measurement, like number of tiles on the floor between them, number of footsteps, etc.
  - Then the same students must move to a distance where just one thumb covers his/her partner's face (with arm held at eye-level and one arm fully extended). They must note this distance using the same unit of measurement.
  - Roles reverse, and activity is repeated.
  - Discussion should include 'what changed'? Did the actual size of your partner's face change? After discussion make sure students understand that the distance changed and therefore the apparent size changed. Relate to common objects like trees that look small from far away, etc.
- Calculations for higher grades:

Students can calculate the small angle formula, relative intensity and the angle subtended by the solar disk from Earth and Pluto and compare. Here are several links that discuss these calculations:

<http://glast.sonoma.edu/teachers/agn/agnposactigpo.pdf>;

<http://astro.uchicago.edu/cara/outreach/resources/ysi94/smallangle.html>

<http://www.starhop.com/High/SolInt-19.pdf>

<http://www.marsacademy.com/amazsp/a4.htm>



- Adaptations for students with disabilities:
  - Seeing impaired: pair these students with other students whom are able to see. It may be useful to compare the size of an object to the sound of something familiar. For example, for a given sound, as the student moves away the sound will become fainter. Likewise, as the student moves away from an object its size will appear smaller, although neither the sound nor the size of an object has actually changed.
  - Hearing impaired: these students should be able to follow the activity by reading the Student Data Sheet as they proceed. Since measurements can be read or indicated using simple hand gestures, there need not be any special considerations for choosing partners with hearing impaired students.
  - Physically impaired: assuming the classroom has been arranged with adequate pathways and stations, students with physical impairments should have no problems completing this activity.

**Interactive:** with the link provided, see what the Sun looks like from Pluto using the **Solar System Simulator!**

- Navigate your browser here: <http://space.jpl.nasa.gov/>
- **Targets and Date**
  - Select the following options from the first two drop down menus: “The Sun” and “Pluto”
  - Choose today’s date or any other date you wish!
- **Field of View**
  - Choose the first round radio button, ‘I want a field of view of [x] degree(s)’
  - Select the degrees from the drop down menu. Begin with 2 or 5 degrees, and then gradually enlarge the field of view.
- You may also wish to see what the Sun looks like from Earth for reference, since this will be quite familiar to students.

## **Standards Addressed:**

### **Benchmarks (AAAS, 1993)**

#### **Chapter 2. The Nature of Mathematics**

##### **2C. Mathematical Inquiry**

Grades 6 through 8

- Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves (for example, a proposition about "all equal-sided triangles" or "all odd numbers").

#### **Chapter 9. The Mathematical World**

##### **9B: Symbolic Relationships**

Grades 6 through 8

- Mathematical statements can be used to describe how one quantity changes when another changes. Rates of change can be computed from differences in magnitudes and vice versa.
- Graphs can show a variety of possible relationships between two variables. As one variable increases uniformly, the other may do one of the following: increase or decrease steadily, increase or decrease faster and faster, get closer and closer to some limiting value, reach some intermediate maximum or minimum, alternately increase and decrease indefinitely, increase or decrease in steps, or do something different from any of these.

#### **Chapter 11. Common Themes**

##### **11B: Models**

Grades 6 through 8

- Models are often used to think about processes that happen too slowly, too quickly, or on too small a scale to observe directly, or that are too vast to be changed deliberately, or that are potentially dangerous.

#### **Chapter 12. Habits of Mind**

##### **12B: Computation and Estimation**

Grades 6 through 8

- Estimate distances and travel times from maps and the actual size of objects from scale drawings.

## **12D: Communication Skills**

Grades 6 through 8

- Organize information in simple tables and graphs and identify relationships they reveal.

## **National Science Education Standards (NRC, 1996)**

### **Content Standards: 5-8**

#### **Science as Inquiry, CONTENT STANDARD A:**

- Abilities necessary to do scientific inquiry

#### **Earth and Space Science, CONTENT STANDARD D:**

- Earth in the solar system

#### **History and Nature of Science, CONTENT STANDARD G:**

- Nature of science

## **Principles and Standards for School Mathematics (NCTM, 2000)**

### **Measurement Standard**

Grades 6 through 8

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements